Quiz 10, MATH 240, Fall 2023

Write your name clearly.

Name:

UID:

(1) Let $\mathbf{w_1} = \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix}$ and $\mathbf{w_2} = \begin{pmatrix} -1 \\ -2 \\ 0 \end{pmatrix}$. Let $S = \{\mathbf{w_1}, \mathbf{w_2}\}$ and let $W = \operatorname{Span} S$.

(a) (6 points) Is S an orthogonal set? Explain your reasoning.

(b) (14 points) Let $\mathbf{v} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$. Express $\mathbf{v}$ as a sum of two vectors, $\mathbf{v} = \operatorname{proj}_W(\mathbf{v}) + \mathbf{v}^\perp$, where $\operatorname{proj}_W(\mathbf{v})$ is in W and $\mathbf{v}^\perp$ is in $W^\perp$. Simplify your algebraic expressions.

(a) $\vec{w}_1 \cdot \vec{w}_2 = -2+2 = 0$, so S is an orthogonal set.

(b) $\operatorname{proj}_W(\vec{v}) = \operatorname{proj}_{\vec{w}_1}(\vec{v}) + \operatorname{proj}_{\vec{w}_2}(\vec{v})$

$= 0 + 0$

$= 0$

So take $\vec{v}^{\perp} = \vec{v}$.

Then $\vec{v} = \vec{0} + \vec{v}^{\perp}$.